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LETTER TO THE EDITOR

Correlation functions in the two-dimensional Ising model with large-scale inhomogeneity caused by a line defect

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Abstract. Exact results are obtained for the local magnetisation and correlation functions of the two-dimensional Ising model with a line defect which produces large-scale inhomogeneity in its vicinity. It is shown that near the critical point the correlation functions exhibit a non-scaling behaviour.

The two-dimensional Ising model with a line defect was proposed by Fisher and Ferdinand (1967) and has been studied by many authors. This model is characterised by the fact that the interaction constants for the nearest-neighbour pair on a single line are changed by an amount λ with respect to their bulk value. Fisher and Ferdinand (1967) found that the incremental specific heat due to this line defect diverges linearly as $T \to T_c$. Bariev (1979) studied the local magnetisation near this line and found the local non-universal behaviour that manifests itself in the fact that the critical exponent β is a continuous function of the microscopic parameter λ $(T < T_c)$

$$\langle S\rangle \sim \tau^{\beta} \qquad \beta = \beta(\lambda) = \frac{1}{2\pi^2}[\cos^{-1}(-\tanh \lambda')]^2 \qquad \lambda' = \frac{2\lambda}{kT_c \sinh(2J/kT_c)} \tag{1}$$

where $\tau = |1 - T/T_c|$ and J is the bulk interaction constant. McCoy and Perk (1980) found that on the defect line the spin–spin correlation function decays as a power with a non-universal exponent η

$$\langle S_{00}S_{N0}\rangle \sim N^{-\eta} \qquad \eta = 2\beta(\lambda). \tag{2}$$

It was shown (Bariev 1979, Fisher 1981) on the basis of a phenomenological approach (Kadanoff and Wegner 1971) that the local non-universality of the model with a line defect is due to the fact that the scale dimensionality of the perturbation Δ_{per} coincides with its physical dimensionality d' $(\Delta_{\text{per}} = d' = 1)$.

Hilhorst and van Leeuwen (1981) and Blöte and Hilhorst (1983) later found the local non-universal behaviour in the semi-infinite Ising model with large-scale inhomogeneity on a boundary. This model is characterised by the fact that the interaction constants close to the boundary are changed with respect to their bulk value. The deviations decay as $-\lambda/n^{\alpha}$ with the distance n from the boundary. In this case the local magnetisation and spin–spin correlation function have a non-universal behaviour on the boundary of system for $\alpha = 1$

$$\langle S\rangle \sim \tau^{\beta_1} \qquad \langle S_{00}S_{N0}\rangle \sim N^{-\eta} \qquad 2\beta = 2\beta(\lambda) = \eta(\lambda) = 1 + \lambda'. \tag{3}$$

The non-universal behaviour in this case was explained by Burkhard (1982) and Gordery (1982).

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Recently Bariev (1988), on the basis of a phenomenological approach, considered the local critical behaviour of the d-dimensional system near the inhomogeneous large-scale internal d'-dimensional defect. The value characterised the strength of the defect decay as $-\lambda/n^{\alpha}$ with the distance n from the defect. In particular this theory predicts that in the case

$$\Delta_{\text{per}} = d - \alpha = d' \tag{4}$$

the local magnetisation and correlation function near the defect have non-scaling behaviour. For the two-dimensional Ising model with inhomogeneity caused by a line defect we have

$$\langle S\rangle \sim \tau^{1/8-\lambda S_{\text{d}} \ln \tau + O(\lambda^2)} \qquad \langle S_{00}S_{N0}\rangle \sim N^{-[1/4+2\lambda S_{\text{d}} \ln N + O(\lambda^2)]} \tag{5}$$

where S_{d} is a constant. In our case $S_{\text{d}} = \lambda'/2\pi\lambda$. The non-scaling form manifests itself in the fact that the local magnetisation has non-power dependence on τ and the correlation function has non-power dependence on distance. In this situation these functions do not have the critical exponents.

Our aim is to test the prediction of the phenomenological theory (5) on the basis of exact results which we obtain for the two-dimensional Ising model with a line defect causing large-scale inhomogeneity.

The Hamiltonian of the model under consideration is given by

$$\mathscr{H} = -\sum_{m,n=-\infty}^{\infty} (J_1(n)S_{mn}S_{m,n+1} + J_2 S_{mn}S_{m+1,n}) \tag{6}$$

where S_{mn} is the spin variable connected with a lattice site having coordinates m and n and taking on values ± 1. The energies of vertical coupling J_2 are constant, but the energies of horizontal coupling are inhomogeneous with the following function of distribution

$$J_1(n) = \begin{cases} J_1 - \lambda(n+1)^{-\alpha} & n \geq 0 \\ J_1 - \lambda(-1)^{-\alpha} & n < 0. \end{cases} \tag{7}$$

Thus provided $\lambda \neq 0$ there is line defect in the zeroth column of the lattice. This defect produces inhomogeneous large-scale variations of the horizontal coupling $J_1(n)$. For $\alpha \to \infty$ the present model coincides with that considered by Bariev (1979) and in the case $J_1(n) = 0$ $(n < 0)$ with the model proposed by Blöte and Hilhorst (1983). In this paper we consider the case $\alpha = 1$ and analyse on the basis of exact results the asymptotic behaviour of local magnetisation and correlation function $\langle S_{00}S_{N0}\rangle$ in the zeroth column of the lattice.

Because the lattice is reflection symmetric about the zeroth column this correlation function can be presented as an $N \times N$ Toeplitz determinant, so that

$$\langle S_{00}S_{N0}\rangle = \det\| a_{jk} \|_{j,k=0}^{N-1} \qquad a_{jK} = a_{j-k} \tag{8}$$

where the elements a_j may be calculated by the methods developed in the earlier papers (McCoy and Wu 1973, Bariev 1979)

$$a_j = (2\pi)^{-1} \int_{-\pi}^{\pi} \mathrm{d}\theta\, \mathrm{e}^{-\mathrm{i}j\theta} \varphi(\mathrm{e}^{\mathrm{i}\theta})$$

$$\varphi(\mathrm{e}^{\mathrm{i}\theta}) = T(\mathrm{e}^{\mathrm{i}\theta})[\varphi_0(\mathrm{e}^{\mathrm{i}\theta}) + k(\mathrm{e}^{\mathrm{i}\theta})][1 + k(\mathrm{e}^{-\mathrm{i}\theta})\varphi_0(^{\mathrm{i}\theta})]^{-1}$$

$$T(\mathrm{e}^{\mathrm{i}\theta}) = (1 - \mathrm{i}S_2\varkappa \sin\theta)(1 + \mathrm{i}S_2\varkappa \sin\theta)^{-1}$$

$$k(\mathrm{e}^{\mathrm{i}\theta}) = -\varkappa(\cos\theta - \mathrm{i}C_2 \sin\theta)(1 - \mathrm{i}S_2\varkappa \sin\theta)^{-1}$$

$$\varphi_0(\mathrm{e}^{\mathrm{i}\theta}) = [(1 - \beta_1 \mathrm{e}^{\mathrm{i}\theta})(1 - \beta_2 \mathrm{e}^{-\mathrm{i}\theta})/(1 - \beta_1 \mathrm{e}^{-\mathrm{i}\theta})(1 - \beta_2 \mathrm{e}^{\mathrm{i}\theta})]^{1/2}$$

$$\begin{aligned}
&\beta_{1,2} = Z_2^{\pm 1}(1-Z_1)(1+Z_1)^{-1} \\
&\varkappa = [1+Z_1^2(1)q(Z_1M_1)^{-1}][1-Z_1^2(1)q(Z_1M_1)^{-1}]^{-1}
\end{aligned} \tag{9}$$

$$Z_i = \tanh K_i \qquad K_i = J_i/kT$$

$$g = -(S_1C_2^* - C_1S_2^* \cosh\eta - S_2^* \sinh\eta)(\sin\theta)^{-1}$$

$$S_i = \sinh 2K_i \qquad C_i = \cosh 2K_i \qquad S_2^* = S_2^{-1}$$

$$C_2^* = C_2S_2^{-1} \qquad \cosh\eta(\theta) = (C_1C_2^* - \cos\theta)(S_1S_2^*)^{-1}$$

$$Z_1(n) = \tanh(J_1(n)/kT).$$

Function Mn is the solution of the recursion equation

$$\begin{aligned}
&M_{n-1} = a + b^2M_n(aM_n + Z_1^2(n))^{-1} \\
&a = -\sin\theta(C_2^* + \cos\theta)^{-1} \qquad b = S_2^*(C_2^* + \cos\theta)^{-1}
\end{aligned} \tag{10}$$

which is in agreement with boundary condition $\lim_{n\to\infty} M_n = -Z_1q$.

Near the critical point T_c the solution of the recursion equation (10) was obtained by Blöte and Hilhorst (1983)

$$M_1^{-1} = -(Z_1q)^{-1}t\Psi(\tfrac{1}{2}+\eta-\nu, 2\mu+1; t)\Psi^{-1}(-\tfrac{1}{2}+\mu-\nu, 2\mu-1; t)$$

$$\mu = (1-\lambda')/2 \qquad \nu = \lambda' t\xi \qquad t = 2(\xi^{-2} + S_{1c}^{-2}\theta^2)^{1/2} \qquad \xi^{-1} = \mathscr{D}\tau \tag{11}$$

$$\mathscr{D} = 2(J_1S_{1c}^{-1} + J_2)/kT_c \qquad \lambda' = 2\lambda/kTS_{1c}$$

where the subscript c means that the function is calculated at the critical point T_c. The degenerate hypergeometric function $\Psi(a, c; t)$ has the different small-t expansions for the different values of parameters a and c. Substituting these different expansions of the function $\Psi(a, c; t)$ in (11) and further in (10) we have opportunity to analyse by the Szegö theorem the magnetisation in the zeroth column of the lattice $\langle S_0\rangle$ for the small τ as a limited value ($N\to\infty$) of the Toeplitz determinant (9). The correlation function (3) may be analysed using the method described by McCoy and Wu (1973) and McCoy and Perk (1980). In the results of this analysis we came to the following conclusions.

(i) $\lambda' < -1$, $\tau\to 0$. In this case the horizontal interactions near the zeroth column are so strong that the magnetisation in the zeroth column remains non-zero as $T\to T_c$:

$$\langle S_0\rangle = B(\lambda) + \mathrm{O}(\tau^{1/2}) \qquad \langle S_{00}S_{N0}\rangle = B^2(\lambda) + \mathrm{O}(N^{-1}). \tag{12}$$

This means that the system near the defect is in the ordered phase. The situation in this case is analogous to that on the inhomogeneous boundary of the semi-infinite Ising model described by Blöte and Hilhorst (1983).

(ii) $\lambda' > 1$, $\tau\to 0$. In this case the interactions near the defect are so weak that the critical properties of the system are analogous to the critical properties near the free boundary of the semi-infinite model (McCoy and Wu 1973, Bariev 1979)

$$\langle S_0\rangle \sim \tau^{1/2} \qquad \langle S_{00}S_{N0}\rangle \sim N^{-1}. \tag{13}$$

(iii) $|\lambda'| < 1$, $\tau\to 0$. In this case the local magnetisation and correlation function have the following forms:

$$\langle S_0\rangle \sim \tau^{\beta(\lambda',\tau)} \qquad \langle S_{00}S_{N0}\rangle \sim N^{-\eta(\lambda',N^{-1})} \tag{14}$$

where

$$\beta(\lambda', \tau) = \frac{1}{2\pi^2}(\cos^{-1} \Delta(\lambda', \tau))^2 \qquad \Delta(\lambda', \tau) = \tanh[\lambda'(\ln \tau + C(\lambda'))]$$

$$C(\lambda') = \ln(2S_{1c}^{-1/2}\mathscr{D}) + \mathbb{C} + O(\lambda') \qquad \mathbb{C} = \text{Euler's constant}$$

$$\eta(\lambda', N^{-1}) = 2\beta(\lambda', N^{-1}).$$

It is easily seen that expression (14) for $\lambda' \to -1$ and $\lambda' \to 1$ takes on the forms (12) and (13) respectively. In the region (3) the local magnetisation and correlation function exhibit the non-scaling behaviour that manifests itself in the fact that in the expression (14) β and η are the continuous functions of τ and N respectively. In this situation it is impossible to introduce the critical exponents that are the basic parameters of the scaling theory. It is easily seen that expression (14) takes on the form (5) in first order in λ'. Thus the results of the phenomenological and rigorous approaches are in agreement.

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